Few-Body Systems (APFB2011) manuscript No.
(will be inserted by the editor)

Xiang Liu · Dian-Yong Chen

Charged bottomonium-like structures $Z_b(10610)$ and $Z_b(10650)$

Received: date / Accepted: date

Abstract The observation of two charged bottomonium-like structures $Z_b(10610)$ and $Z_b(10650)$ has stimulated extensive studies of the properties of $Z_b(10610)$ and $Z_b(10650)$. In this talk, we briefly introduce the research status of $Z_b(10610)$ and $Z_b(10650)$ combined with our theoretical progress.

Keywords Bottomonium-like state · Exotic state · Initial single pion emission

1 Introduction

In the past 8 years, experiments have made big progress on the observations of charmonium-like or bottomonium-like state X, Y, Z, which have stimulated theorists' interest in revealing their underlying structures. Thus, studying charmonium-like and bottomonium-like states is an active and important research field in hadron physics at present, which can further deepen our understanding of the properties of X, Y, Z and improve our knowledge about non-perturbative QCD.

Very recently, the Belle Collaboration [1] reported the first observation of two charged bottomonium-like structure, which makes the family of bottomonium-like abundant. By analyzing the $\Upsilon(nS)\pi^{\pm}$ $(n = 1, 2, 3)$ and $h_b(mP)\pi^{\pm}$ $(m = 1, 2)$ invariant mass spectra of $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$, $h_b(mP)\pi^+\pi^-$ hidden-bottom dipion decays, Belle observed that there exist two enhancement structures around 10610 MeV and 10650 MeV, which are named as $Z_b(10610)$ and $Z_b(10650)$, where $Z_b(10610)$ and $Z_b(10650)$ are close to the thresholds of $B\bar{B}^*$ and $B^*\bar{B}^*$ respectively.

Due to the peculiarities of $Z_b(10610)$ and $Z_b(10650)$, theorists have paid more attentions to the observed novel phenomena by different approaches [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. In the following, we will introduce the theoretical progress on the study of $Z_b(10610)$ and $Z_b(10650)$.

2 The puzzles in the hidden-bottom dipion decays of $\Upsilon(5S)$ and Z_b structures

Before the observation of two Z_b structures [1], the Belle Collaboration measured the $e^+e^- \to \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$ processes near the peak of the $\Upsilon(5S)$ resonance at $\sqrt{s} = 10.87$ GeV [15], which indicates that there exist the anomalous $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$ productions, i.e, the branching ratios of $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ are larger than the dipion-transition rates between the lower members of the Υ family by two orders of magnitude [15].

Xiang Liu
School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
E-mail: xiangliu@lzu.edu.cn

Dian-Yong Chen
Nuclear Theory Group, Institute of Modern Physics of CAS, Lanzhou 730000, China

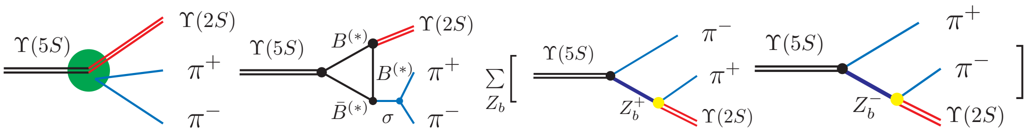

Fig. 1 (Color online.) The schematic diagrams of $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$. The first diagram describes the direct production of $\Upsilon(2S)\pi^+\pi^-$ without the intermediated meson contribution. The second diagram is due to the rescattering mechanism [16], where dipion is from intermediate $\sigma(600)$. The third and the fourth diagrams reflect two newly observed Z_b structures contributing to the $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ decay.

For solving this puzzling phenomena, rescattering mechanism was proposed in Ref. [16]. Since $\Upsilon(5S)$ is above the threshold of B meson pair, the coupled channel effect becomes important, which makes the intermediated hadronic loops constructed by $B^{(*)}/\bar{B}^{(*)}$ mesons play crucial role to understanding the anomalous $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$ production [16]. As an alternative explanation, tetraquark state $Y_b(10890) = [bq][\bar{b}\bar{q}]$ was introduced in Ref. [17]. Later, authors of Ref. [18] studied the Belle data by analyzing the dipion invariant mass spectrum and the $\cos\theta$ distribution of Y_b decays into $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$, and claimed that the tetraquark interpretation can well describe the anomalous rates observed by Belle [15].

Later, in Ref. [19] we indicated that the $\cos\theta$ distribution of $Y_b \to \Upsilon(2S)\pi^+\pi^-$ given by Ref. [18] is not consistent with the Belle data, and proposed an alternative approach to try to explain Belle observation of $\Upsilon(5S)$ decays into $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$, where the interference between the direct dipion transition and the final state interaction corresponding to the first and the second diagrams of Fig. 1, respectively. Under this scenario, we can explain the anomalous rates of $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$ production in $\Upsilon(5S)$ decays, especially the inverse rates $\Gamma(\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-) > \Gamma(\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-)$. Besides, the dipion invariant spectrum and the $\cos\theta$ distribution of $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$ can be reproduced. However, the $\cos\theta$ distribution of $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ cannot be described by the scenario in Ref. [19].

This fact mentioned above shows that a new puzzle appears in the $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ decay, which cannot be understood by the tetraquark state picture [18] or rescattering mechanism [19]. Thus, we need to consider new mechanism involved in the $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ decay.

Since two Z_b structures are from the hidden-bottom dipion decays of $\Upsilon(5S)$ [15], we realized that there exists the extra intermediate $Z_b(10610)$ and $Z_b(10650)$ contributions to the $\Upsilon(5S)$ decays just depicted by the last two diagrams in Fig. 1. Thus, in Ref. [3] we included all mechanisms listed in Fig. 1 to redo the analysis of the $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ decay.

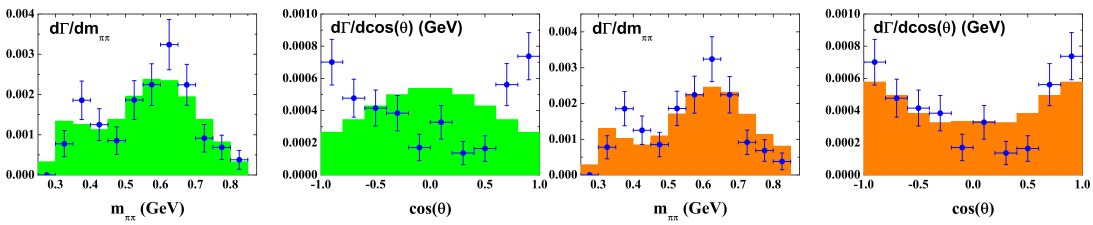

Fig. 2 (Color online.) The dipion invariant mass spectrum and the $\cos\theta$ distribution of the $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ decay. Here, the dots with error bars are from Belle measurement, while the histograms are the theoretical results [3]. The first two diagrams or the last two diagrams are the results without or with the intermediate Z_b contribution to the $\Upsilon(2S)\pi^+\pi^-$ decay.

The results presented in Fig. 2 indicate that including the intermediate $Z_b(10610)$ and $Z_b(10650)$ contribution can produce the $\cos\theta$ distribution of $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ consistent with the experimental data [15]. This observation provides an indirect evidence to the existence of two charged Z_b structures, and gives a possible approach to solve the puzzles existing in the $\Upsilon(5S)$ hidden-bottom dipion decays. However, we must find the source to generate the $Z_b(10610)$ and $Z_b(10650)$ structures. In the following, we introduce the exotic state explanations to $Z_b(10610)$ and $Z_b(10650)$.

3 Exotic state assignments to two charged Z_b structures

Since the charged $Z_b(10610)$ and $Z_b(10650)$ are close to the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds, respectively, $Z_b(10610)$ and $Z_b(10650)$ can be as good candidate of exotic state. Before the observation of these two charged bottomonium-like states, the analysis in Refs. [20, 21] indicates that there probably exists a loosely bound S-wave $B\bar{B}^*$ molecular state, where the One-Boson-Exchange (OBE) model is applied to the dynamical calculation.

For further understanding the structure of $Z_b(10610)$ and $Z_b(10650)$, different theoretical groups have performed the study of $Z_b(10610)$ and $Z_b(10650)$ considering different exotic state assignments to $Z_b(10610)$ and $Z_b(10650)$, which mainly include molecular states composed of $B^{(*)}\bar{B}^{(*)}$ mesons [2, 4, 6, 10, 12, 13], and hidden-bottom tetraquark states [7, 9, 14].

By the OBE model, we systematically calculated the interaction between $B^{(*)}$ and $\bar{B}^{(*)}$ mesons, where π, ρ, ω, σ meson exchanges are introduced when deducing the effective potential of $B^{(*)}\bar{B}^{(*)}$ molecular systems. If $Z_b(10610)$ and $Z_b(10650)$ are $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states respectively, we can construct their flavor wave functions [20, 21], $|Z_b(10610)^{\pm}\rangle = \frac{1}{\sqrt{2}}\big(|B^{*\pm}\bar{B}^0\rangle + |B^{\pm}\bar{B}^{*0}\rangle\big)$ and $|Z_b(10610)^0\rangle = \frac{1}{2}\Big[\big(|B^{*+}B^-\rangle - |B^{*0}\bar{B}^0\rangle\big) + \big(|B^+B^{*-}\rangle - |B^0\bar{B}^{*0}\rangle\big)\Big]$ for $Z_b(10610)$, $|Z_b(10650)^0\rangle = \frac{1}{\sqrt{2}}\big(|B^{*+}B^{*-}\rangle - |B^{*0}\bar{B}^{*0}\rangle\big)$ and $|Z_b(10650)^{\pm}\rangle = |B^{*\pm}\bar{B}^{*0}\rangle$ for $Z_b(10650)$.

Under the Breit approximation, the effective potential of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems can be related to the scattering amplitude

$$\mathcal{V}_E^{B^{(*)}\bar{B}^{(*)}}(\mathbf{q}) = -\frac{\mathcal{M}(B^{(*)}\bar{B}^{(*)} \to B^{(*)}\bar{B}^{(*)})}{\sqrt{\prod_i 2M_i \prod_f 2M_f}},$$

where M_i and M_j denote the masses of the initial and final states respectively. Thus, we obtain the potential in the coordinate space $\mathcal{V}(\mathbf{r})$ by performing the Fourier transformation

$$\mathcal{V}_E^{B^{(*)}\bar{B}^{(*)}}(\mathbf{r}) = \int \frac{d\mathbf{p}}{(2\pi)^3}\, e^{i\mathbf{p}\cdot\mathbf{r}} \mathcal{V}_E^{B^{(*)}\bar{B}^{(*)}}(\mathbf{q})\mathcal{F}^2(q^2, m_E^2), \tag{1}$$

where the monopole form factor (FF) $\mathcal{F}(q^2, m_E^2) = (\Lambda^2 - m_E^2)/(\Lambda^2 - q^2)$ is introduced, which reflects the structure effect of the vertex of the heavy mesons interacting with the light mesons. m_E denotes the exchange meson mass. We consider both S-wave and D-wave interactions between $B^{(*)}$ and $\bar{B}^{(*)}$ mesons, which make the $B\bar{B}^*$ and $B^*\bar{B}^*$ to be further expressed as

$$\big|Z_{B\bar{B}^*}^{(\alpha)\;(\prime)}\big\rangle = \begin{pmatrix} \big|BB^*({}^3S_1)\big\rangle \\ \big|BB^*({}^3D_1)\big\rangle \end{pmatrix}, \; \big|Z_{B^*\bar{B}^*}^{(\alpha)}[0]\big\rangle = \begin{pmatrix} \big|B^*\bar{B}^*({}^1S_0)\big\rangle \\ \big|B^*\bar{B}^*({}^5D_0)\big\rangle, \end{pmatrix}$$

$$\big|Z_{B^*\bar{B}^*}^{(\alpha)}[1]\big\rangle = \begin{pmatrix} \big|B^*\bar{B}^*({}^3S_1)\big\rangle \\ \big|B^*\bar{B}^*({}^3D_1)\big\rangle \\ \big|B^*\bar{B}^*({}^5D_1)\big\rangle \end{pmatrix}, \; \big|Z_{B^*\bar{B}^*}^{(\alpha)}[2]\big\rangle = \begin{pmatrix} \big|B^*\bar{B}^*({}^5S_2)\big\rangle \\ \big|B^*\bar{B}^*({}^1D_2)\big\rangle \\ \big|B^*\bar{B}^*({}^3D_2)\big\rangle \\ \big|B^*\bar{B}^*({}^5D_2)\big\rangle \end{pmatrix} \tag{2}$$

with $\alpha = S, T$, where we use the notation ${}^{2S+1}L_J$ to denote the total spin S, angular momentum L, total angular momentum J of the $B\bar{B}^*$ or $B^*\bar{B}^*$ system. Indices S and D indicate that the couplings between B^* and $\bar{B}^*$ occur via the S-wave and D-wave interactions, respectively.

In general, the total effective potentials of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems are

$$V_{\mathrm{Total}}^{Z_{B\bar{B}^*}^{(\alpha)\;(\prime)}} = \Big\langle Z_{B\bar{B}^*}^{(\alpha)\;(\prime)}\Big| \sum_{E=\pi,\eta,\sigma,\rho,\omega} \mathcal{V}_E^{B\bar{B}^*}(r)\big|Z_{B\bar{B}^*}^{(\alpha)\;(\prime)}\big\rangle, \tag{3}$$

$$V_{\mathrm{Total}}^{Z_{B^*\bar{B}^*}^{(\alpha)}[\mathrm{J}]} = \Big\langle Z_{B^*\bar{B}^*}^{(\alpha)}[\mathrm{J}]\Big| \sum_{E=\pi,\eta,\sigma,\rho,\omega} \mathcal{V}_E^{B^*\bar{B}^*}(r)\big|Z_{B^*\bar{B}^*}^{(\alpha)}[\mathrm{J}]\big\rangle, \tag{4}$$

Table 1 The obtained bound state solutions (binding energy E and root-mean-square radius r_{RMS}) for the $Z_b(10610)$ and $Z_b(10650)$ systems [10].

state	Λ (GeV)	E (MeV)	r_{RMS} (fm)	state	Λ (GeV)	E (MeV)	r_{RMS} (fm)
	2.1	-0.22	3.05		2.2	-0.81	1.38
$Z_b(10610)$	2.3	-1.64	1.31	$Z_b(10650)$	2.4	-3.31	0.95
	2.5	-4.74	0.84		2.8	-14.94	0.52

which are 2×2 and $(J+2)\times(J+2)$ matrices, respectively (see Ref. [10] for more details).

The numerical result listed in Table 1 indicates that $Z_b(10610)^{\pm}$ and $Z_b(10650)^{\pm}$ can be explained as the $B^*\bar{B}$ and $B^*\bar{B}^*$ molecular states [10].

4 Initial single pion emission mechanism

Besides explaining $Z_b(10610)$ and $Z_b(10650)$ under the exotic state assignments, we also explored whether there exist other mechanisms resulting in $Z_b(10610)$ and $Z_b(10650)$ enhancement structures, which is an interesting research topic. Since the observation of the $Z_b(10610)$ and $Z_b(10650)$ structures is from the hidden-bottom dipion decays of $\Upsilon(5S)$, studying the decay mechanisms existing in the $\Upsilon(5S)$ decays can provide some hints to understand $Z_b(10610)$ and $Z_b(10650)$.

Just presented in Sect. 3, there are at least three different decay mechanisms just shown in Fig. 1. Apart from the first two mechanisms corresponding to the first two diagrams of Fig. 1, we noticed that the last two diagrams in Fig. 1 can be naturally replacements as the diagrams in Fig. 3, where the emitted single pion directly from $\Upsilon(5S)$ makes the intermediate $B^{(*)}\bar{B}^{(*)}$ meson pair with low momenta, which interact with each other to transit into final states by exchanging $B^{(*)}$ mesons. This decay mechanism is named as the Initial Single Pion Emission (ISPE) mechanism in Ref. [11].

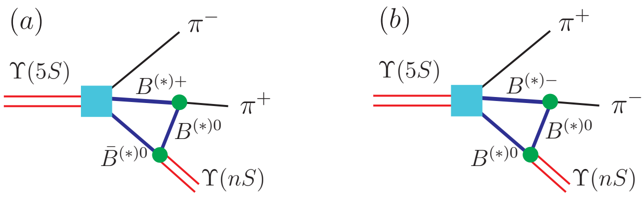

Fig. 3 (Color online.) The schematic diagrams describing the ISPE mechanism of $\Upsilon(5S)$. Here, we use $\Upsilon(5S)\to\Upsilon(nS)\pi^+\pi^-$ as an example.

By the ISPE mechanism [11], we studied the line shapes of $\frac{d\Gamma(\Upsilon(5S\to\Upsilon(nS)\pi^+\pi^-))}{dm_{\Upsilon(nS)\pi^+}}$ $(n=1,2,3)$ and $\frac{d\Gamma(\Upsilon(5S\to h_b(mP)\pi^+\pi^-))}{dm_{h_b(mP)\pi^+}}$ $(m=1,2)$. We found sharp structures around 10610 MeV and 10650 MeV in the obtained theoretical line shapes of $\frac{d\Gamma(\Upsilon(5S\to\Upsilon(nS)\pi^+\pi^-))}{dm_{\Upsilon(nS)\pi^+}}$ and $\frac{d\Gamma(\Upsilon(5S\to h_b(mP)\pi^+\pi^-))}{dm_{h_b(mP)\pi^+}}$ distributions, which could naturally correspond to the $Z_b(10610)$ and $Z_b(10650)$ structures newly observed by Belle [1]. In Fig. 4, we shown the numerical result in the scenario of ISPE.

If the ISPE mechanism is a universal mechanism existing the decay of heavy flavor quarkonia, we extend this mechanism to study the hidden-charm dipion decays of higher charmonia [22]. We predicted the charged charmonium-like structures around the $D\bar{D}^*$ and $D^*\bar{D}^*$ thresholds. Since these novel phenomena are accessible by the BESIII, Belle, BaBar experiments, and Belle-II or SuperB, we suggest future experiments to carry out the search for these charged charmonium-like structures.

5 Summary

Recently two charged bottomonium-like structures $Z_b(10610)$ and $Z_b(10650)$ were announced by the Belle Collaboration [1], which inspired theorists' interest in understanding their structure and under-

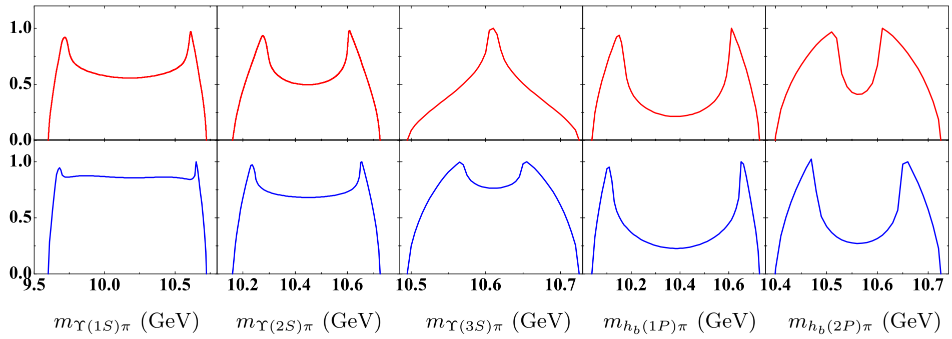

Fig. 4 (Color online.) The invariant mass spectra of $\Upsilon(nS)\pi^{\pm}$ $(n = 1, 2, 3)$ and $h_b(mP)\pi^{\pm}$ $(m = 1, 2)$ of $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$ and $\Upsilon(5S) \to h_b(mP)\pi^+\pi^-$ decays.

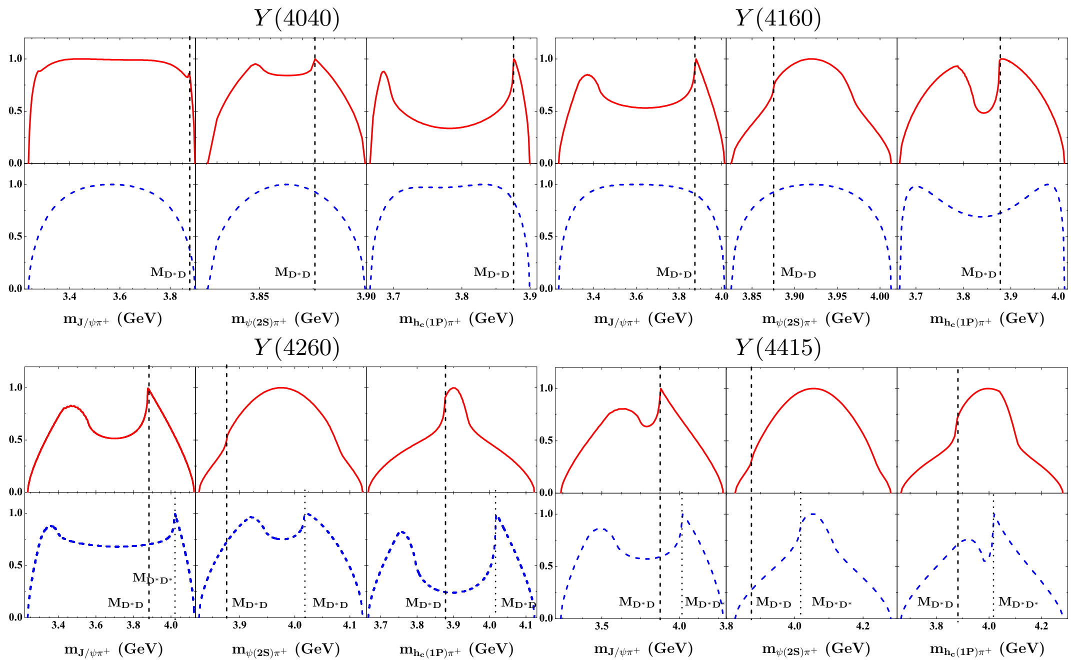

Fig. 5 (Color online.) The predicted invariant mass spectra of $J/\psi\pi^+$, $\psi(2S)\pi^+$ and $h_c(1P)\pi^+$ for the $\psi(4040)$, $\psi(4160)$, $\psi(4415)$ and $Y(4260)$ decays into $J/\psi\pi^+\pi^-$, $\psi(2S)\pi^+\pi^-$ and $h_c(1P)\pi^+\pi^-$. Here, the solid, dashed correspond to the results considering intermediate $D\bar{D}^* + h.c.$ and $D^*\bar{D}^*$ respectively in Fig. 3. The vertical dashed lines and the dotted lines denote the threshhold of $D^*\bar{D}$ and $D^*\bar{D}^*$ respectively [11]. Here, the maximum of the line shape is normalized to 1.

lying mechanism behind these novel phenomena. At present, different explanations to $Z_b(10610)$ and $Z_b(10650)$ were proposed due to the peculiarities of $Z_b(10610)$ and $Z_b(10650)$.

We briefly introduce the research progress on the study of $Z_b(10610)$ and $Z_b(10650)$. Especially, we emphasize our theoretical work. In Ref. [3], we indicated that two Z_b structures are important to understand the anomalous $\cos\theta$ distribution of $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$. $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular state assignments provide the possible explanation to the structure of $Z_b(10610)$ and $Z_b(10650)$. In addition, we found the ISPE mechanism existing $\Upsilon(5S)$ hidden-bottom dipion decays [22], which results in the enhancement structures around the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds. This observation also provides a possibility to understand $Z_b(10610)$ and $Z_b(10650)$. What is more important it that we also predicted charged charmonium-like structures close to the $D\bar{D}^*$ and $D^*\bar{D}^*$ thresholds. The further experimental search for these structures will be helpful to test the ISPE mechanism proposed in Ref. [22].

In conclusion, we still need to pay more theoretical and experimental efforts to reveal the underlying mechanism relevant to the charged bottomonium-like structures $Z_b(10610)$ and $Z_b(10650)$.

6 Acknowledgement

X.L. would like to thank the organizers of APFB2011 conference, especially Prof. Hyun-Chul Kim for his warm invitation. X.L. also thank Lanzhou University for financial support (from the 985 Project). We enjoy the collaboration with Prof. Shi-Lin Zhu, Prof. Xue-Qian Li, Dr. Jun He, Zhi-Gang Luo and Zhi-Feng Sun. This project is partly supported by the National Natural Science Foundation of China under Grants No. 11175073, No 11005129, No. 11035006, No. 11047606; the Ministry of Education of China (FANEDD under Grant No. 200924, DPFIHE under Grant No. 20090211120029, NCET under Grant No. NCET-10-0442, the Fundamental Research Funds for the Central Universities); and the West Doctoral Project of Chinese Academy of Sciences.

References

1. Adachi, I. et al. [Belle Collaboration]: Observation of two charged bottomonium-like resonances. arXiv:1105.4583 [hep-ex]
2. Bondar A.E., Garmash A., Milstein A.I., Mizuk R., Voloshin M.B.: Heavy quark spin structure in Z_b resonances. arXiv:1105.4473 [hep-ph]
3. Chen D.Y., Liu X., Zhu S.L.: Charged bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$ and the $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ decay. arXiv:1105.5193 [hep-ph]
4. Zhang J.R., Zhong M., Huang M.Q.: Could $Z_b(10610)$ be a $B^*\bar{B}$ molecular state?. arXiv:1105.5472 [hep-ph]
5. Bugg D.V.: An Explanation of Belle states $Z_b(10610)$ and $Z_b(10650)$. arXiv:1105.5492 [hep-ph]
6. Voloshin M.B.: Radiative transitions from $\Upsilon(5S)$ to molecular bottomonium. arXiv:1105.5829 [hep-ph]
7. Yang Y.C., Ping J.L., Deng C., Zong H.S.: Dynamical study of the $Z_b(10610)$ and $Z_b(10650)$ as molecular states. arXiv:1105.5935 [hep-ph]
8. Nieves J., Valderrama M.P.: Deriving the existence of $B\bar{B}^*$ bound states from the $X(3872)$ and Heavy Quark Symmetry. arXiv:1106.0600 [hep-ph]
9. Guo T., Cao L., Zhou M.Z., Chen H.: The Possible candidates of tetraquark: $Z_b(10610)$ and $Z_b(10650)$. arXiv:1106.2284 [hep-ph]
10. Sun Z.F., He J., Liu X., Luo Z.G., Zhu S.L.: $Z_b(10610)^\pm$ and $Z_b(10650)^\pm$ as the $B^*\bar{B}$ and $B^*\bar{B}^*$ molecular states. Phys. Rev. D **84**, 054002 (2011)
11. Chen D.Y., Liu X.: $Z_b(10610)$ and $Z_b(10650)$ structures produced by the initial single pion emission in the $\Upsilon(5S)$ decays. arXiv:1106.3798 [hep-ph]
12. Cleven M., Guo F.K., Hanhart C., Meissner U.G: Bound state nature of the exotic Z_b states. arXiv:1107.0254 [hep-ph]
13. Cui C.Y., Liu Y.L., Huang M.Q.: Investigating different structures for the $Z_b(10610)$. arXiv:1107.1343 [hep-ph]
14. Navarra F.S., Nielsen M., Richard J.M.: Z_b is not X_b. Is that a question?. arXiv:1108.1230 [hep-ph]
15. Chen K.F. et al. [Belle Collaboration]: Observation of anomalous Upsilon(1S) pi+ pi- and Upsilon(2S) pi+ pi- production near the Upsilon(5S) resonance. Phys. Rev. Lett. **100**, 112001 (2008)
16. Meng C., Chao K.T.: Scalar resonance contributions to the dipion transition rates of Upsilon(4S,5S) in the re-scattering model. Phys. Rev. D **77**, 074003 (2008)
17. Ali A., Hambrock C., Ahmed I., Aslam M.J.: A Case for hidden $b\bar{b}$ tetraquarks based on $e^+e^- \to b\bar{b}$ cross section $\sqrt{s} = 10.54$ and 11.20 GeV. Phys. Lett. B **684**, 28 (2010)
18. Ali A., Hambrock C., Aslam M.J.: A Tetraquark interpretation of the Belle data on the anomalous $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$ production near the $\Upsilon(5S)$ resonance. Phys. Rev. Lett. **104**, 162001 (2010) [Erratum-ibid. **107**, 049903 (2011)]
19. Chen D.Y., He J., Li X.Q., Liu X.: Dipion invariant mass distribution of the anomalous $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$ production near the peak of $\Upsilon(10860)$
20. Liu Y.R., Liu X., Deng W.Z., Zhu S.L.: Is $X(3872)$ Really a Molecular State?. Eur. Phys. J. **C56**, 63-73 (2008)
21. Liu X., Luo Z.G., Liu Y.R., Zhu S.L.: $X(3872)$ and Other Possible Heavy Molecular States. Eur. Phys. J. **C61**, 411-428 (2009)
22. Chen D.Y., Liu X.: Predicted charged charmonium-like structures in the hidden-charm dipion decay of higher charmonia. Phys. Rev. **D84**, 034032 (2011)